Examiners' Report
Principal Examiner Feedback

Summer 2019

Pearson Edexcel International A Level
In Further Pure Mathematics F1 (WFM01/01)

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General Introduction

This paper was accessible, and most candidates could find plenty of opportunities to demonstrate their knowledge and understanding in this paper. There were some testing questions involving complex numbers, mathematical induction and coordinate geometry that allowed good discrimination between the higher grades.

In summary, Q1, Q2, Q3, Q4, Q5(a) and Q6 were a good source of marks for the average student, mainly testing standard ideas and techniques and Q5(b), Q5(c), Q7, Q8, Q9(a) and Q9(b) were discriminating at the higher grades. Q9(c) proved to be the most challenging question on the paper.

Question 1

Q1 proved accessible with most candidates scoring full marks.

In Q1(a), most candidates gave a correct $\mathrm{f}'(x)$. A few candidates incorrectly differentiated $-\dfrac{7}{2x}$ to give either $-\dfrac{7}{2x^2}$, $\dfrac{7}{4x^2}$ or $\dfrac{14}{x^2}$.

In Q1(b), many candidates found the values of $\mathrm{f}(0.5)$ and $\mathrm{f}'(0.5)$ before proceeding to apply these to a correct Newton-Raphson formula. A lack of working did mean that it was sometimes difficult for examiners to determine whether Newton-Raphson was applied correctly. For example, a few candidates stated the general formula followed by their final answer while others just stated their final answer with no working.

In Q1(c), most candidates correctly evaluated $\mathrm{f}(3)$ and $\mathrm{f}(3.5)$, with many indicating that one was positive and the other negative. Although, most referred to a sign change, not all made an explicit conclusion that this implied a root in the interval $[3, 3.5]$.

In Q1(d), many candidates drew a diagram and most used similar triangles to form a correct equation in β and proceeded to solve it correctly. Some candidates, however, formed an equation in β with one of their fractions the wrong way around, while others used an odd number of negative lengths in their working. Some candidates obtained an answer outside the interval $[3.5, 3.6]$, with many not realising that their answer should have been enough evidence that something was amiss. A few candidates went back to first principles and found the equation of the line joining the points $(3, 3.83...)$ and $(3.5, -4.16...)$, before proceeding to find where the line crossed the x-axis.

Question 2

Most candidates found a correct $\det(\mathbf{M})$ in terms of k and equated the result to the ratio of the two given areas. The majority used the only the positive ratio, 16, thereby only obtaining one quadratic equation in k which was usually solved correctly to give

$k = 14, -2$. Only a minority applied a negative $\det(\mathbf{M})$ to form the second quadratic equation in *k*, with many of them solving it correctly to give $k = 6+4\sqrt{2}, 6-4\sqrt{2}$.

Some candidates rejected correct values for *k* such as $k = -2, 6+4\sqrt{2}, 6-4\sqrt{2}$, in the mistaken assumption that these were not real.

Question 3

In Q3(i), most candidates started by multiplying the numerator and denominator of the right hand side of the equation $z^* - 3z = \frac{5\text{i}}{3-\text{i}}$ by $(3+\text{i})$ to give $\frac{5\text{i}(3+\text{i})}{(3-\text{i})(3+\text{i})}$. Whilst most obtained a correct $\frac{-5+15\text{i}}{10}$, a few incorrect denominators such as $9-1$ or $3+1$ were seen. Many candidates applied $z = a+b\text{i}$ and $z^* = a-b\text{i}$ to $z^* - 3z$. They usually obtained $-2a-4b\text{i}$ and proceeded to find the values of *a* and *b* by equating both the real and imaginary parts of the equation $-2a-4b\text{i} = -\frac{1}{2}+\frac{3}{2}\text{i}$. A few candidates applied an alternative method of multiplying both sides of the equation $-2a-4b\text{i} = \frac{5\text{i}}{3-\text{i}}$ by $(3-\text{i})$ to give $(-2a-4b\text{i})(3-\text{i}) = 5\text{i}$. These candidates usually formed and solved simultaneous equations to find values for *a* and *b*.

In Q3(i), some candidates simplified $z^* - 3z$ incorrectly to give either $-2z$, $-4z$ or $2\,\text{Re}(z)$, while other candidates did not leave their final answer as a complex number, *z*.

In Q3(ii)(a), some candidates did not realise that the required answer was in the second quadrant and many of them found an angle in either the first or third quadrants. Most candidates worked in radians, but a few gave their answer in degrees. Those candidates who plotted $-4+5\text{i}$ on an Argand diagram were usually more successful in obtaining the correct answer.

In Q3(ii)(b), only a minority of candidates deduced the correct $k = 4$. It was clear that many candidates did not think geometrically in order to understand that $\arg(w+k) = \frac{\pi}{2}$ leads to the real part of $-4+5\text{i}+k$ being equal to 0.

In Q3(ii)(c), most candidates gave a correct solution. Some candidates did not apply Pythagoras' Theorem correctly to find the magnitude of the expression $|w+c\text{i}|$. Various incorrect methods were seen such as simplifying $|-4+5\text{i}+c\text{i}| = 4\sqrt{5}$ to give either $16-(5+c)^2 = 80$, $16+(5+c)^2 = 4\sqrt{5}$ or $16+25+c^2 = 80$. After applying Pythagoras' Theorem correctly, most candidates solved the resulting quadratic to give $c = -13, 3$. A few candidates rejected $c = -13$ for no clear reason.

Question 4

Q4 was accessible to most candidates.

In Q4(a), most candidates wrote $\sum_{r=1}^{3k}(4r+1)$ as $4.\frac{1}{2}(3k)(3k+1)+3k$ and used algebra to achieve the correct result $9k(2k+1)$. A few candidates applied $\sum_{r=1}^{n}(4r+1)$ to give $4.\frac{1}{2}(n)(n+1)+n$ which they simplified to give $2n^2+3n$. They usually proceeded to achieve the correct result by deducing that $\sum_{r=1}^{3k}(4r+1)=2(3k)^2+3(3k)$.

In Q4(b), most candidates used the standard result for $\sum_{r=1}^{n}r^2$ and their answer to Q4(a) to deduce the equation $2.\frac{1}{6}k(k+1)(2k+1)=9k(2k+1)$. Only a minority of candidates, then cancelled $k(k+1)$ from both sides to achieve a linear equation in *k*. The approach taken by most candidates was to multiply out both sides of their equation and rearrange to achieve a cubic equation in *k*. Many solved their cubic equation to obtain either 2 or 3 values for *k*, namely 0, -0.5 and 26. Most candidates went onto conclude that only one solution $k=26$ satisfied the given equation.

Question 5

Q5 discriminated well between candidates of all abilities.

In Q5(a), most candidates recognised that matrix **A** represented a rotation with some candidates not always stating that centre of rotation. Most candidates stated the correct angle and sense of rotation with a few incorrectly stating $120°$ anti-clockwise or $60°$ clockwise.

Q5(b) discriminated well. The most successful candidates were those who deduced the correct answer by considering $\mathbf{A}^6$ as a succession of rotations. Some candidates, who were usually less successful, proceeded to multiply matrices multiple times in order to calculate $\mathbf{A}^6$.

In Q5(c), some candidates found the matrix **C** by correctly applying $\mathbf{B}^{-1}\mathbf{A}$. A few candidates applied an incorrect $\mathbf{B}^{-1}=\frac{1}{\det(\mathbf{B})}\mathbf{B}$ or evaluated $\det(\mathbf{B})$ incorrectly as $30+28$. A minority attempted $\mathbf{AB}^{-1}$ after writing down the incorrect matrix equation

$\mathbf{A} = \mathbf{CB}$. A few candidates stated a correct $\mathbf{A} = \mathbf{BC}$ and used $\mathbf{C} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ to find the elements a, b, c and d by solving simultaneous equations. Arithmetic errors or sign errors sometimes led to a loss of marks in Q5(c).

Question 6

Only a few candidates found and applied α, β = $\frac{-1+\sqrt{31}\,\mathrm{i}}{4}$, $\frac{-1-\sqrt{31}\,\mathrm{i}}{4}$ in Q6. These candidates lost a considerable number of marks because they did not obey the instruction 'Without solving the (quadratic) equation' which was stated in this question.

In Q6(a), most candidates stated the correct values for both $\alpha+\beta$ and $\alpha\beta$. A few candidates, who did not divide the given equation $2x^2+x+4=0$ by 2, incorrectly stated $\alpha+\beta$ as -1 and $\alpha\beta$ as 4.

In Q6(b), most candidates correctly substituted their values for $\alpha+\beta$ and $\alpha\beta$ into their correct expressions for $\alpha^2+\beta^2$ and $\alpha^3+\beta^3$. Some candidates made bracketing or sign errors in this part while a few candidates used incorrect expressions for $\alpha^3+\beta^3$.

The complete method in Q6(c) was understand by most candidates, but a considerable number failed to score full marks because they made substitution, manipulation, or bracketing errors. Some candidates applied $\left(x-\left(\alpha^3+\frac{1}{\beta}\right)\right)\left(x-\left(\beta^3+\frac{1}{\alpha}\right)\right)=0$, while most found the values for the sum and product of $\left(\alpha^3+\frac{1}{\beta}\right)$ and $\left(\beta^3+\frac{1}{\alpha}\right)$. A few candidates did not simplify their sum $\alpha^3+\beta^3+\frac{1}{\beta}+\frac{1}{\alpha}$ to become $\alpha^3+\beta^3+\frac{\alpha+\beta}{\alpha\beta}$ before substituting in their values for $\alpha+\beta$ and $\alpha\beta$. Most candidates proceeded to use a correct method to form the quadratic equation described in the question. The three main errors in establishing the required quadratic equation were: applying the incorrect method of x^2 + (sum)x + (product)$=0$; the omission of '$=0$'; and the failure to give integer coefficients.

Question 7

In Q7(a), almost all candidates wrote down the complex conjugate root $-1+3\mathrm{i}$.

In Q7(b), most candidates used the conjugate pair to write down and multiply out $(z-(-1-3\mathrm{i}))(z-(-1+3\mathrm{i}))$ in order to identify the quadratic factor $z^2+2z+10$, while other candidates achieved this quadratic factor by using the sum and product of roots method. Some candidates struggled to find the second quadratic factor due to the unknown values of a and b in the given quartic $\mathrm{f}(z)=z^4-6z^3+az^2-44z+b$.. Some candidates created extra work for themselves in using various methods to determine the values of a and b. Of those candidates who found a second quadratic factor, most used long division, although some compared coefficients or used inspection with a number making sign or manipulation errors at this stage. Those who obtained a second quadratic factor usually applied a correct method of completing the square or using the quadratic formula to find the other two complex roots.

Question 8

In Q8, many candidates successfully showed that $\mathrm{f}(n) = 3^{4n-2} + 2^{6n-3}$ is divisible by 17 for $n = 1$. There were varying approaches to induction with finding $\mathrm{f}(k+1) - \mathrm{f}(k)$ or directly finding $\mathrm{f}(k+1)$ being the most popular. There were other valid methods that candidates employed with varying degrees of success, such as an attempt to find $\mathrm{f}(k+1)$ in terms of k and M, where $\mathrm{f}(k) = 17M = 3^{4k-2} + 2^{6k-3}$, or an attempt to find $\mathrm{f}(k+1) - m\mathrm{f}(k)$ with a suitable value for m. Although many candidates wrote down a correct expression for either $\mathrm{f}(k+1) - \mathrm{f}(k)$ or $\mathrm{f}(k+1)$, some did not manipulate their expression to a correct result for $\mathrm{f}(k+1)$ of the form $64(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$ or $64\mathrm{f}(k) + 17(3^{4k-2})$ or equivalent.

Some candidates did not bring all strands of their proof together to give a fully correct proof. A minimal acceptable proof, following on from completely correct work, would incorporate the following: assuming the general result is true for $n = k$; then showing the general result is true for $n = k+1$; showing the general result is true for $n = 1$; and finally concluding that the general result is true for all positive integers.

Question 9

Q9 discriminated well between candidates of all abilities.

In Q9(a), candidates used a variety of methods to find $\frac{\mathrm{d}y}{\mathrm{d}x}$, with the most common being to make y the subject in order to find $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{a}}{\sqrt{x}}$. Other methods seen included implicit differentiation or the chain rule with parametric equations. Most candidates used $P(ap^2, 2ap)$ to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of p, followed by a complete method for finding the equation of the normal, l, to the parabola C at P. At this stage, a significant minority of candidates, struggled to make any further creditable progress. Of those that did, most used the information that the point B also lied on l and substituted $x = 10a$, $y = 0$ into their equation for l and found a value for p. Most candidates found the correct coordinates $P(8a, 4\sqrt{2}a)$ by substituting their correct $p = 2\sqrt{2}$ into $P(ap^2, 2ap)$. There were a number of manipulation errors seen in Q9(a) including a common cancelling error which led to an incorrect $p = 8$.

In Q9(a), a few candidates who did not use a calculus method but just stated that the gradient of the normal to *C* at *P* is $-p$ lost the first three marks.

In Q9(b), many candidates identified the focus as $S(a,\ 0)$. Those who drew a diagram of triangle *SBP* were often more successful in applying a correct method to find its area. Most candidates found the area by applying the formula $\frac{1}{2}(\text{base})(\text{height})$ with a few applying a determinant formula. Some erroneously used a base length of $10a-0$ rather than $9a$ (i.e. $10a-a$).

Q9(c) proved to be the most challenging question on the paper, and only the most able candidates were able to a produce fully correct solution. Many candidates left Q9(c) blank or made no creditable progress.

The most popular method was to substitute their line *l* into the equation of the circle. Some candidates either gave up prior to or at the point of reaching the quadratic equation $36x^2-720ax+3591a^2=0$, while others proceeded to solve this quadratic equation and so find a correct $R(9.5a,\ \sqrt{2}\,a)$. A few candidates then used their $P(8a,\ 4\sqrt{2}\,a)$ from Q9(a) in a Pythagoras method to find the distance *PR*. Common mistakes with this method included bracketing errors, manipulation errors and a few candidates who stated the length *PR* as $9.5a$, which is really the *x*-coordinate of *R*.

Only a few candidates, who usually drew a clear diagram, found *PR* by applying the complete method $PR=PB-1.5a=\sqrt{(10a-8a)^2+(0-4\sqrt{2}a)^2}-1.5a$. Some candidates used the incorrect method of applying $PR=PB+1.5a$.